2014 年全国硕士研究生招生考试试题

一、选择题(本题共8小题,每小题4分,共32分.在每小题给出的四个选项中,只有一项符合题目要求,把所选项前的字母填在题后的括号内.)

(1) 当$x\to 0^+$时,若$\ln^\alpha(1+2x)$,$(1-\cos x)^{\frac{1}{\alpha}}$均是比$x$高阶的无穷小量,则$\alpha$的取值范围是(　　)

(A) $(2,+\infty)$.　　(B) $(1,2)$.　　(C) $\left(\frac{1}{2},1\right)$.　　(D) $\left(0,\frac{1}{2}\right)$.

(2) 下列曲线中有渐近线的是(　　)

(A) $y=x+\sin x$.　　(B) $y=x^2+\sin x$.　　(C) $y=x+\sin\frac{1}{x}$.　　(D) $y=x^2+\sin\frac{1}{x}$.

(3) 设函数$f(x)$具有2阶导数,$g(x)=f(0)(1-x)+f(1)x$,则在区间$[0,1]$上,(　　)

(A) 当$f'(x)\geqslant 0$时,$f(x)\geqslant g(x)$.　　(B) 当$f'(x)\geqslant 0$时,$f(x)\leqslant g(x)$.

(C) 当$f''(x)\geqslant 0$时,$f(x)\geqslant g(x)$.　　(D) 当$f''(x)\geqslant 0$时,$f(x)\leqslant g(x)$.

(4) 曲线$\begin{cases}x=t^2+7,\\y=t^2+4t+1\end{cases}$上对应于$t=1$的点处的曲率半径是(　　)

(A) $\frac{\sqrt{10}}{50}$.　　(B) $\frac{\sqrt{10}}{100}$.　　(C) $10\sqrt{10}$.　　(D) $5\sqrt{10}$.

(5) 设函数$f(x)=\arctan x$.若$f(x)=xf'(\xi)$,则$\lim\limits_{x\to 0}\frac{\xi^2}{x^2}=$(　　)

(A) 1.　　(B) $\frac{2}{3}$.　　(C) $\frac{1}{2}$.　　(D) $\frac{1}{3}$.

(6) 设函数$u(x,y)$在有界闭区域D上连续,在D的内部具有2阶连续偏导数,且满足$\frac{\partial^2 u}{\partial x\partial y}\neq 0$及$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$,则(　　)

(A) $u(x,y)$的最大值和最小值都在D的边界上取得.

(B) $u(x,y)$的最大值和最小值都在D的内部取得.

(C) $u(x,y)$的最大值在D的内部取得,最小值在D的边界上取得.

(D) $u(x,y)$的最小值在D的内部取得,最大值在D的边界上取得.

(7) 行列式$\begin{vmatrix}0&a&b&0\\a&0&0&b\\0&c&d&0\\c&0&0&d\end{vmatrix}=$(　　)

(A) $(ad-bc)^2$.　　(B) $-(ad-bc)^2$.　　(C) $a^2d^2-b^2c^2$.　　(D) $b^2c^2-a^2d^2$.

(8) 设$\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\alpha}_3$均为3维向量,则对任意常数$k,l$,向量组$\boldsymbol{\alpha}_1+k\boldsymbol{\alpha}_3,\boldsymbol{\alpha}_2+l\boldsymbol{\alpha}_3$线性无关是向量组$\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\alpha}_3$线性无关的(　　)

(A) 必要非充分条件.　　(B) 充分非必要条件.

(C) 充分必要条件.　　(D) 既非充分也非必要条件.

二、填空题(本题共 6 小题,每小题 4 分,共 24 分,把答案填在题中横线上.)

(9) $\int_{-\infty}^{1} \frac{1}{x^2+2x+5}dx =$ ______.

(10) 设 $f(x)$ 是周期为 4 的可导奇函数,且 $f'(x)=2(x-1), x\in[0,2]$,则 $f(7)=$ ______.

(11) 设 $z=z(x,y)$ 是由方程 $e^{2yz}+x+y^2+z=\frac{7}{4}$ 确定的函数,则 $dz\Big|_{\left(\frac{1}{2},\frac{1}{2}\right)}=$ ______.

(12) 曲线 L 的极坐标方程是 $r=\theta$,则 L 在点 $(r,\theta)=\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ 处的切线的直角坐标方程是______.

(13) 一根长度为 1 的细棒位于 x 轴的区间 $[0,1]$ 上,若其线密度 $\rho(x)=-x^2+2x+1$,则该细棒的质心坐标 $\bar{x}=$ ______.

(14) 设二次型 $f(x_1,x_2,x_3)=x_1^2-x_2^2+2ax_1x_3+4x_2x_3$ 的负惯性指数为 1,则 a 的取值范围是______.

三、解答题(本题共 9 小题,共 94 分,解答应写出文字说明、证明过程或演算步骤.)

(15)(本题满分 10 分)

求极限 $\lim\limits_{x\to+\infty}\frac{\int_1^x\left[t^2\left(e^{\frac{1}{t}}-1\right)-t\right]dt}{x^2\ln\left(1+\frac{1}{x}\right)}$.

(16)(本题满分 10 分)

已知函数 $y=y(x)$ 满足微分方程 $x^2+y^2y'=1-y'$,且 $y(2)=0$,求 $y(x)$ 的极大值与极小值.

(17)(本题满分 10 分)

设平面区域 $D=\{(x,y)\mid 1\leqslant x^2+y^2\leqslant 4, x\geqslant 0, y\geqslant 0\}$. 计算 $\iint\limits_D \frac{x\sin(\pi\sqrt{x^2+y^2})}{x+y}dxdy$.

(18)（本题满分 10 分）

设函数$f(u)$具有 2 阶连续导数，$z=f(e^x\cos y)$满足

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=(4z+e^x\cos y)e^{2x}.$$

若$f(0)=0, f'(0)=0$，求$f(u)$的表达式.

(19)（本题满分 10 分）

设函数$f(x)$，$g(x)$在区间$[a, b]$上连续，且$f(x)$单调增加，$0\leqslant g(x)\leqslant 1$. 证明：

（Ⅰ）$0\leqslant \int_a^x g(t)\,dt\leqslant x-a,\ x\in[a, b]$；

（Ⅱ）$\int_a^{a+\int_a^b g(t)\,dt} f(x)\,dx\leqslant \int_a^b f(x)g(x)\,dx$.

(20)（本题满分 11 分）

设函数$f(x)=\frac{x}{1+x}, x\in[0, 1]$. 定义函数列：

$$f_1(x)=f(x),\quad f_2(x)=f(f_1(x)),\quad \cdots,\quad f_n(x)=f(f_{n-1}(x)),\quad \cdots.$$

记S_n是由曲线$y=f_n(x)$，直线$x=1$及x轴所围平面图形的面积. 求极限$\lim\limits_{n\to\infty} nS_n$.

(21)（本题满分 11 分）

已知函数 $f(x,\ y)$ 满足 $\dfrac{\partial f}{\partial y}=2(y+1)$，且 $f(y,\ y)=(y+1)^2-(2-y)\ln y$，求曲线 $f(x,\ y)=0$ 所围图形绕直线 $y=-1$ 旋转所成旋转体的体积.

(22)（本题满分 11 分）

设矩阵 $\boldsymbol{A}=\begin{pmatrix}1 & -2 & 3 & -4\\ 0 & 1 & -1 & 1\\ 1 & 2 & 0 & -3\end{pmatrix}$，$\boldsymbol{E}$ 为 3 阶单位矩阵.

（Ⅰ）求方程组 $\boldsymbol{Ax}=\boldsymbol{0}$ 的一个基础解系；

（Ⅱ）求满足 $\boldsymbol{AB}=\boldsymbol{E}$ 的所有矩阵 $\boldsymbol{B}$.

(23)（本题满分 11 分）

证明 n 阶矩阵 $\begin{pmatrix}1 & 1 & \cdots & 1\\ 1 & 1 & \cdots & 1\\ \vdots & \vdots & & \vdots\\ 1 & 1 & \cdots & 1\end{pmatrix}$ 与 $\begin{pmatrix}0 & \cdots & 0 & 1\\ 0 & \cdots & 0 & 2\\ \vdots & & \vdots & \vdots\\ 0 & \cdots & 0 & n\end{pmatrix}$ 相似.

2014

1.B

$$\lim_{x\,o\,0^+}\frac{\ln\ (1\quad 2x)}{x}\quad \lim_{x\,o\,0}\frac{(2x)^{D}}{x}\quad 2\ \lim_{x\,o\,0}x^{D\quad 1}\quad 0$$

$$\lim_{x\,o\,0}\frac{(1\quad \cos x)^{\frac{1}{2}}}{x}\quad \lim_{x\,o\,0}\frac{(\frac{1}{2}x^2)^{\frac{1}{D}}}{x}\quad (\frac{1}{2})^{\frac{1}{D}}\lim_{x\,o\,0}x^{\frac{2}{a}\quad 1}\quad 0$$

$$?\,\frac{2}{D}\quad 1\,!\,0\,?\ D\quad 2$$

2　C

$$y\quad x\quad \sin\frac{1}{x}$$

$$k\quad \lim_{x\,o\,f}\frac{y}{x}\quad \lim_{x\,o\,f}\frac{x\quad \sin\frac{1}{x}}{x}\quad 1$$

$$\lim_{x\,o\,f}\ y\quad x\quad \lim_{x\,o\,f}\sin\frac{1}{x}\quad 0$$

$$?\,y\quad x\quad \sin\frac{1}{x}\qquad\qquad y\quad x$$

3　D

$$f\ (x\quad)x^2,\qquad [0,1]$$

$$f(0)\quad 0$$

$$f(1)\quad 1$$

$$?\,g\ (x\quad)0\ \tilde{}(1\quad x)\quad 1\ \tilde{x}\quad x$$

$$f\ (x?\ )g\ \ x\ \ (\quad)d$$

$$f\,''\ (x\quad)2\ \ t\ 0\ ?\ D$$

4　C

$$\frac{dy}{dx} \quad \frac{2t \quad 4}{2t}$$

$$\frac{dy}{dx}\Big|_{t \ 1} \quad 3$$

$$\frac{d^2y}{dx^2} \quad \frac{\frac{2 \ 2t \quad 2(2t \quad 4)}{(2t)^2}}{2t} \quad \frac{8}{(2t)^3}$$

$$? \ \frac{d^2y}{dx^2}\Big|_{t \ 1} \quad 1$$

$$k \quad \frac{|y''|}{(1 \quad y'^2)^{\frac{3}{2}}} \quad \frac{1}{(1 \quad 3^2)^{\frac{3}{2}}}$$

$$? \ R \quad \frac{1}{k} \quad (1 \quad 3^2)^{\frac{3}{2}} \quad 10^{\frac{3}{2}} \quad 10\sqrt{10}$$

5 D

$$\frac{f(x)}{x} \quad \frac{\arctan x}{x} \quad \frac{1}{1 \quad \xi^2} . \quad \xi^2 \quad \frac{x \quad \arctan x}{\arctan x} .$$

$$\lim_{x \ o \ 0} \frac{\xi^2}{x^2} \quad \lim_{x \ o \ 0} \frac{x \quad \arctan x}{x^2 \ \text{[redacted]}} \quad \lim_{x \ o \ 0} \frac{x \quad \arctan x}{x^3}$$

$$\lim_{x \quad 0} \frac{1 \quad \frac{1}{1 \quad x^2}}{3x^2} \quad \lim_{\alpha \quad 0} \frac{x^2}{3x^2(1 \quad x^2)} \quad \frac{1}{3} .$$

6 A

$$B \quad \frac{w^2 u}{wxwy} \quad 0 \quad w \qquad \frac{{}^2u}{x^2} \quad \frac{w^2u}{wy^2} \quad 0 \qquad A \quad \frac{w^2u}{wx^2} \quad B \quad \frac{w^2u}{wy^2} \qquad .$$

$$AC \quad B^2 \quad 0 \qquad u(x, y) \qquad D \qquad .$$

$$D \qquad .$$

7 B

$$\begin{vmatrix} 0 & a & b & 0 \\ a & 0 & 0 & b \\ 0 & c & d & 0 \\ c & 0 & 0 & d \end{vmatrix}$$

$$= a \times (-1)^{2+1} \begin{vmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & d \end{vmatrix} + c \times (-1)^{4+1} \begin{vmatrix} a & b & 0 \\ 0 & 0 & b \\ c & d & 0 \end{vmatrix}$$

$$= -a \times d \times (-1)^{3+3} \begin{vmatrix} a & b \\ c & d \end{vmatrix} - c \times b \times (-1)^{2+3} \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= -ad \begin{vmatrix} a & b \\ c & d \end{vmatrix} + bc \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= (bc - ad) \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= -(ad - bc)^2$$

8 A

$\varepsilon_1, \varepsilon_2, \varepsilon_3$

$\alpha_1(\varepsilon_1 + k\varepsilon_3) + \alpha_2(\varepsilon_2 + l\varepsilon_3) + \alpha\varepsilon = 0$

$\alpha_1\varepsilon_1 + \alpha_2\varepsilon_2 + (\alpha_1 k + l\alpha_2 + \alpha)\varepsilon_3 = 0$

$\Rightarrow \alpha_1 = \alpha_2 = k\alpha_1 + l\alpha_2 + \alpha = 0$

$\varepsilon_1 + k\varepsilon_3, \ \varepsilon_2 + l\varepsilon_3, \ \varepsilon$

$\varepsilon_1 + k\varepsilon_3, \ \varepsilon_2 + l\varepsilon_3, \ \varepsilon \qquad \varepsilon_1, \varepsilon_2, \varepsilon_3$

$$\varepsilon_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \varepsilon_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \varepsilon_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

9. $\int_{-\infty}^{1} \frac{1}{x^2 + 2x + 5} dx = \int_{-\infty}^{1} \frac{1}{(x+1)^2 + 4} dx = \frac{1}{2} \arctan \frac{x+1}{2} \Big|_{-\infty}^{1} = \frac{1}{2}\left[\frac{\pi}{4} - \left(-\frac{\pi}{2}\right)\right] = \frac{3}{8}\pi$

10.

$f'\ x \quad 2(x \quad 1)x \bullet [0,2]$

$? \ f(x) \quad x^2 \quad 2x \quad c$

$f(x)$

$? \ f(0) \quad 0 \ ? \ c \quad 0$

$? \ f(x) \quad x^2 \quad 2x$

$x \bullet [0,2]$

$f(x) \qquad 4$

$? \ f(7) \quad f(3) \quad f(\ 1) \quad f(1) \quad (1 \quad 2) \quad 1$

11 x

$$e^{2yz}(2y \sim \frac{z}{x}) \quad 2x \quad \frac{wz}{wx} \quad 0$$

$$x \quad \frac{1}{2}, y \quad \frac{1}{2}$$

$$\frac{wz}{wx} = \frac{1}{e^{z \ \frac{1}{2} \frac{1}{2}} + 1}$$

y

$$e^{2yz}(2z \quad 2y \frac{z}{y}) \quad 2y \quad \frac{wz}{wy} \quad 0$$

$$x \quad \frac{1}{2}, y \quad \frac{1}{2}$$

$$\frac{wz}{wy} = \frac{1 \quad z \quad \frac{1}{2} \quad \frac{1}{2} \quad e^{z \ \frac{1}{2} \frac{1}{2}}}{e^{z \ \frac{1}{2} \frac{1}{2}} + 1}$$

12.

$$\begin{cases} x = r\cos\theta = \theta\cos\theta \\ y = r\sin\theta = \theta\sin\theta \end{cases}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sin\theta + \theta\cos\theta}{\cos\theta - \theta\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{1 + \frac{\pi}{2}\cdot 0}{0 - \frac{\pi}{2}\cdot 1} = -\frac{2}{\pi}$$

$$\theta = \frac{\pi}{2} \quad \begin{cases} x = \cos\theta = 0 \\ y = \sin\theta = \frac{\pi}{2} \end{cases}$$

$$\left(y - \frac{\pi}{2}\right) = -\frac{2}{\pi}(x - 0)$$

$$y = -\frac{2}{\pi}x + \frac{\pi}{2}$$

13

$$\frac{\int_0^1 x f(x)dx}{\int_0^1 f(x)dx} = \frac{\int_0^1 3x(x^2 - 2x + 1)dx}{\int_0^1 3(x^2 - 2x + 1)dx} = \frac{\left(\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2\right)\Big|_0^1}{\left(\frac{1}{3}x^3 - x^2 + x\right)\Big|_0^1} = \frac{11}{20}$$

14

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 2ax_1x_3 + 4x_2x_3$$

$$= (x_1 + ax_3)^2 + (x_2 + 2x_3)^2 + 4x_3^2 - a^2x_3^2$$

[redacted] 1

$4-a^2$? 0 t

? 2 d a d2

15.

$$\lim_{x\,o\,f}\frac{\int_1^x (t^2(e^{\frac{1}{t}}\quad 1)\quad t)\,dt}{x^2\ln(1\quad \frac{1}{x})}\quad \lim_{x o\, f}\frac{3_1^x (t^2(e^{\frac{1}{t}}\quad 1)\quad t)\,dt}{x^2\quad \frac{1}{x}}\quad \lim_{\alpha\, f}\frac{x^2(e^{\frac{1}{x}}\quad 1)\quad x}{1}\quad \lim_{o f} x^2(e^{\frac{1}{x}3}\quad 1\quad \frac{1}{x})$$

$$\underline{\underline{\frac{1}{x}}}\quad t\lim_{x\,o\,f}\frac{e^t\quad 1\quad t}{t^2}\quad \lim_{x o\, f}\frac{1\quad t\quad \frac{1}{2}t^2\quad 2(t^2)\quad 1\quad t}{t^2}\quad \frac{1}{2}$$

16

[redacted] $y'\quad 1\quad y'$

$y?\quad \frac{1\quad x^2}{y^2\quad 1}$

$y'\quad 0,\ ?\ x\quad r1$

$y?\quad \frac{2x(y^2\quad 1)\quad (1\quad x^2)\ \tilde{}\ 2yy'}{(y^2\quad 1)^2}$

[redacted] $y'(\quad 1)\quad 0$

$?\ y''(1)\quad \frac{2}{y^2(1)\quad 1}\ \text{Ɑ}0,\quad ?y(1)$

$y''(\quad 1)\quad \frac{2}{y^2(1)\quad 1}\ ^2 0,\ y(\quad 1)$

[redacted] $\frac{1\quad x^2}{y^2\quad 1},\ ?\ (y^2\quad 1)dy\quad (1\quad x^2)dx, ?\quad (y^2\quad 1)dy\quad 3(1\quad x^2)dx$

$?\ \frac{1}{3}y^3\quad y\quad x\quad \frac{1}{3}x^3\quad c$

$y(2)\quad 0$

$c?\quad \frac{2}{3}$

$?\ \frac{1}{3}y^3\quad y\quad x\quad \frac{1}{3}x^3\quad \frac{2}{3}$

$x \quad 1$

$? \frac{1}{3} y^3(1) \quad y(1) \quad 1 \quad \frac{1}{3} \quad \frac{2}{3}$

$y(1)? \quad 1$

$x \quad 1,$

$? \frac{1}{3} y^3(\quad 1) \quad y(\quad 1) \quad 1 \quad \frac{1}{3} \quad \frac{2}{3} \quad 0$

$? \, y(\quad 1) \quad 0$

17

$D \quad y \quad x$

$$3\,3_{D} \frac{x \sin(\sqrt{x^2 \quad y^2})}{x \quad y} dxdy \qquad 3\,3_{D} \frac{y \sin(S \sqrt{x^2 \quad y^2})}{x \quad y} dxdy$$

$$\frac{1}{2} \; 3\,3_{D} \frac{x \sin(\sqrt{x^2 \quad y^2})}{x \quad y} \quad \frac{y \sin(S \sqrt{x^2 \quad y^2})}{x \quad y} dxdy$$

$$\frac{1}{2} \; 3\,3_{D} \sin(S\sqrt{x^2 \quad y^2}) dxdy$$

$$\frac{1}{2} \; 3_0^{\frac{S}{2}} d\, T \; 3_1^2 \sin(\; Sr) r \, \mathrm{d} r \qquad \frac{1}{4} \; 3_1^2 r d \cos(\; rS)$$

$$\frac{1}{4} r \cos(\quad r) |_1^2 \qquad \frac{1}{4} \; 3_1^2 \cos(S \; r) \, \mathrm{d} r$$

$$\frac{1}{2} \quad \frac{1}{4} \qquad \frac{3}{4}$$

18

$\frac{wz}{wx}$ f' ~e^x ~cos y ,

$\frac{w^2z}{wx^2}$ cos y ~(f'' ~e^xcos~ y e^x~ f' e^x) ~ f''(e^xcos~) ~y^2 f' e^xcos ~ y

$\frac{wz}{wy}$ f' ~e^x (~ siny),

$\frac{w^2z}{wy^2}$ e^x[f'' ~e^x(~siny) cos ~] y( ) e^xsiny f'' ^{2} cos f' ~ y e^x ~

$\frac{^2z}{x^2}$ $\frac{w^2z}{wy^2}$ f'' ~e^{2x} w(4 z e^xcos~)y e^{2x}
w

f''(e^xc os y) 4 f(e^x~cos y) e^x cos~ y

t e^x ~cos y, ? f''( ) t 4 f( ) t t

? y '' 4 y x

$\hat{O}$ 4 0 ? x r2 ? y(x) $C_1 e^{2x}$ $C_2 e^{2x}$

y (ax b) ? y - $\frac{1}{4}$x

? y y(x) y $C_1 e^{2x}$ $C_2 e^{2x}$ $\frac{1}{4}$x

y(0)=0= C_1 C_2

y'(0)=0=x $_1$ x $_2$ $\frac{1}{4}$

-C_1 C_2 0 ° C_1 $\frac{1}{16}$

2C_1 2C_2 $\frac{1}{4}$? ® ° C_2 $\frac{1}{16}$

? f( ) $\frac{1}{16}e^2$ $\frac{1}{16}e^{2P}$ $\frac{1}{4}P$ P

19.

I

$h_1(x) \quad \int_a^x g(t)dt$

$h_1(a) \quad 0$

$h_1'(x) \quad g(x) \quad 0 \qquad t$

$?\ h_1(x)$

$? \quad x \bullet > a,b @ \quad h_1(x)\ t\ 0$

$h_2(x) \quad \int_a^x g(t)dt \quad x \quad a$

$h_2'(x) \quad g(x) \quad 1$

[redacted] $)\ d1 \quad ?h_2'(x) \quad d0$

$h_2(x) \qquad h_2(a)? \quad 0$

$? \quad x \bullet > a,b @ \quad h_2(x)\ d\ 0$

$$p(x) \quad \int_a^x f(u)g(u)du \quad \int_a^{a \int_a^x g(t)dt} f(u)du \qquad 3$$

$$p'(x) \quad f(x)g(x) \quad f[a \quad \int_a^x g(t)dt]\ \tilde{}\ g(x) \qquad f(x) \quad f[a \quad \int_a^x g(t)dt]^a g(x)$$

[redacted] $)d \ \ 1 \qquad d$

$$? \ \int_a^x g(t)dt \ d \ \int_a^x dt \quad x \quad a \ ?\ a \quad \int_a^x g(t)dt \ d\ x$$

$f(x)$

$$?\ f(x)\ t\ \ f[a \quad \int_a^x g(t)dt]\ ?\ \ p'(x)\ t\ \ 0$$

$?\ p(x)$

$p \ \ a \ =0 \ ?\ p(b) \ \ t\ 0$

$$\int_a^b f(x)g(x)dx \ \ t \int_a^{a \int_a^b g(t)dt} f(x)dx \qquad 3$$

$f(x) = \frac{x}{1+x}$, $f_1(x) = f(x)$

$$f_2(x) = f(f_1(x)) = \frac{\frac{x}{1+x}}{1+\frac{x}{1+x}} = \frac{x}{1+2x}$$

$$f_3(x) = f(f_2(x)) = \frac{\frac{x}{1+2x}}{1+\frac{x}{1+2x}} = \frac{x}{1+3x}$$

$$f_n(x) = \frac{x}{1+nx},\ x \in [0,1]$$

$$S_n = \int_0^1 \frac{x}{1+nx}\,dx = \frac{1}{n}\int_0^1 \frac{nx+1-1}{1+nx}\,dx$$

$$= \frac{1}{n}\int_0^1 \left(1-\frac{1}{1+nx}\right)dx$$

$$= \frac{1}{n} - \frac{1}{n^2}\ln(1+n)$$

$$\lim_{n\to\infty} nS_n = \lim_{n\to\infty} n\left[\frac{1}{n} - \frac{1}{n^2}\ln(1+n)\right] = 1 - \lim_{n\to\infty}\frac{\ln(1+n)}{n}$$

$$= 1$$

21.

$$\frac{\partial f}{\partial y} = 2(y+1)$$

$$f(x,y) = y^2 + 2y + \varphi(x)$$

$$f(y,y) = (y+1)^2 - (2-y)$$

$$f(y,y) = y^2 + 2y + \varphi(y)$$

$$\varphi(y) = y - 1$$

$$f(x,y) = y^2 + 2y + x - 1$$

$$f(x,y) = 0 \Rightarrow x = -y^2 - 2y + 1$$

$$V = \int_0^2 \pi[f(x)+1]^2\,dx = \pi\int_0^2 [f^2(x) + 2f(x) + 1]\,dx = \pi\int_0^2 (2-x)\,dx = \pi\left(2x - \frac{x^2}{2}\right)\Big|_0^2 = 2\pi$$

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$$(A)=\begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 1 & 1\\ 1 & 2 & 0 & 3\end{pmatrix} \xrightarrow{r_1 \; r_3} \begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 1 & 1\\ 0 & 4 & 3 & 1\end{pmatrix} \xrightarrow{4r_2 \; r_3} \begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 1 & 1\\ 0 & 0 & 1 & 3\end{pmatrix}$$

$$\xrightarrow[3r_3 \; r_1]{r_3 \; r_2} \begin{pmatrix}1 & 2 & 0 & 5\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & 3\end{pmatrix} \xrightarrow{2r_2 \; r_1} \begin{pmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & 3\end{pmatrix}$$

$$\begin{matrix}x_1 & x_4\\ x_2 & 2x_4\\ x_3 & 3x_4\\ x_4 & x_4\end{matrix} \qquad \begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix} c \begin{pmatrix}1\\ 2\\ 3\\ 1\end{pmatrix} \quad c$$

$$B=\begin{pmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{pmatrix}$$

$$A\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} \begin{pmatrix}1\\ 0\\ 0\end{pmatrix} \quad \begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 1 & 1\\ 1 & 2 & 0 & 3\end{pmatrix}$$ [redacted]

$$A\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix} \begin{pmatrix}0\\ 1\\ 0\end{pmatrix} \quad \begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 1 & 1\\ 1 & 2 & 0 & 3\end{pmatrix}$$ [redacted]

$$A\begin{pmatrix}z_1\\ z_2\\ z_3\end{pmatrix} \begin{pmatrix}0\\ 0\\ 1\end{pmatrix} \quad \begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 1 & 1\\ 1 & 2 & 0 & 3\end{pmatrix}$$ [redacted]

$$\begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 1 & 1\\ 1 & 2 & 0 & 3\end{pmatrix}$$ [redacted]

$$\to \begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 1 & 1\\ 0 & 0 & 1 & 3\end{pmatrix}$$ [redacted]

$$\to \begin{pmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & -3\end{pmatrix}$$ [redacted] $$\begin{pmatrix}-1\\ 1\\ 1\end{pmatrix}$$

$$x_1 \quad 1 \quad 2 \qquad y_1 \quad 1 \quad 5 \qquad z_1 \quad 1 \quad 1$$
$$x_2 \quad 2 \quad 1 \qquad y_2 \quad 2 \quad 3 \qquad z_2 \quad 2 \quad 1$$
$$x_3 = c_1 \quad 3 \quad 1 \qquad y_3 = c_2 \quad 3 \quad 4 \qquad z_3 = c_3 \quad 3 \quad 1$$
$$x_4 \quad 1 \quad 0 \qquad y_4 \quad 1 \quad 0 \qquad z_4 \quad 1 \quad 0$$

$$B = \begin{pmatrix} c_1 & 2 & c_2 & 6 & c_3 & 1 \\ 2c_1 & 1 & 2c_2 & 3 & 2c_3 & 1 \\ 3c_1 & 1 & 3c_2 & 4 & 3c_3 & 1 \\ c_1 & & c_2 & & c_3 & \end{pmatrix}$$

c_1, c_2, c_3

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A: 1 1 [redacted] / 1 1 [redacted] / [redacted] / 1 1 [redacted]

B: 0 0 [redacted] 1 / 0 0 [redacted] 2 / [redacted] / 0 0 [redacted] n

$|\lambda E - A|$ = | λ 1 1 [redacted] / 1 λ 1 [redacted] / [redacted] / 1 1 [redacted] 1 | $(\lambda - n)\lambda^{n-1}$

A n λ_1=n, λ_2= [redacted] 0

A A

n

A [redacted] 0 [redacted] 0 $|\lambda E - B|$ = | λ 0 [redacted] 1 / 0 λ [redacted] 2 / [redacted] / 0 0 [redacted] N | $(\lambda - n)\lambda^{n-1}$

B n λ_1=n, λ_2= [redacted] 0

$|0E - B|$ = | 0 0 [redacted] / 0 0 [redacted] / [redacted] / 0 0 [redacted] |

r(0 E B) 1

B n-1 0 n-1

n a

0 «

B B [redacted] «

[redacted]

«

0

A B